Statistics

Fall 2022

Lecture 22

Feb 19-8:47 AM

Constructing Confidence Interval for population Proportion p:

$\hat{p}$ → p-hat

Sample Proportion

Point-estimate

$\hat{p} = \frac{x}{n}$

$\hat{q} = 1 - \hat{p}$

$x = n\hat{p}$ if decimal ⇒ Round-up

Ans: $\quad < p <$

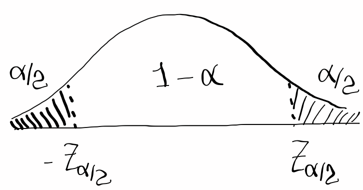
Format: $\hat{p} - E < p < \hat{p} + E$

Margin of error

$$E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

Critical Value

$(1-\alpha)\cdot 100\%$ Conf. level



If α not given ⇒ Use .05

If C-level not given ⇒ Use 95% = .95

Nov 30-6:02 AM

In a Survey of 400 students, 20% of them had a full-time job.

$\hookrightarrow n=400$ $\quad \hookrightarrow \hat{p}=.2$

$x=n\hat{p}=400(.2)=\boxed{80}$

Find 90% Confidence interval for the prop. of all students that have a full-time job.

$\hookrightarrow$ C-level: .9

$\hat{p}-E<p<\hat{p}+E$

$.2-.03<p<.2+.03$

$E=Z_{\alpha/2}\cdot\sqrt{\frac{\hat{p}\hat{q}}{n}}$

$=1.645\cdot\sqrt{\frac{(.2)(.8)}{400}}$

$Z_{.05}=\text{invNorm}(.95,0,1)=1.645$

$E=\boxed{.03}$

$\boxed{.17<p<.23}$

We are 90% confident that between 17% and 23% of all students have a full-time job.

Now using TI:

STAT TESTS 1-PropZInt

$x=80$

$n=400$

C-level: .9

Calculate

$.1671<p<.2329$

$\boxed{.17<p<.23}$

$\hat{p}=\frac{.23+.17}{2}=.2$

$E=\frac{.23-.17}{2}=.03$

Nov 30-6:09 AM

In a Survey of 180 nurses, 25% of them were Carpooling to work.

$\hookrightarrow n=180$ $\quad \hookrightarrow \hat{p}=.25$

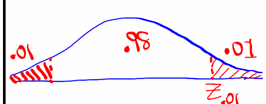
$x=n\hat{p}=180(.25)=\boxed{45}$

Find 98% Conf. interval for the prop. of all nurses that carpool to work.

$\hookrightarrow$ C-level: .98

$\hat{p}-E<p<\hat{p}+E$

$.25-.08<p<.25+.08$



$E=Z_{\alpha/2}\cdot\sqrt{\frac{\hat{p}\hat{q}}{n}}$

$Z_{.01}=\text{invNorm}(.99,0,1)=\boxed{2.326}=2.326\cdot\sqrt{\frac{(.25)(.75)}{180}}\approx\boxed{.08}$

We are 98% confident that between 17% and 33% of all nurses carpool to work.

$\boxed{.17<p<.33}$

Using TI:

STAT TESTS 1-PropZInt

$x=45$

$n=180$

C-level: .98

Calculate

$.17492<p<.32508$

$\boxed{.17<p<.33}$

$\hat{p}=\frac{.33+.17}{2}=\boxed{.25}$

$E=\frac{.33-.17}{2}=\boxed{.08}$

Nov 30-6:22 AM

In a Survey of 375 LA residents, 210 of them were fan of LA Lakers. $\rightarrow n=375$ $\rightarrow x=210$

$\hat{p} = \frac{x}{n} = \frac{210}{375} = \boxed{.56}$ Point-estimate Sample Proportion

Find confidence interval for the prop. of all LA residents that are fan of LA Lakers.

$\rightarrow$ NO C-level $\Rightarrow$ use .95

1-PropZInt

$x=210$

$n=375$

C-level: .95

$.50976 < p < .61024$

$\boxed{.51 < p < .61}$

we are 95% confident that between 51% to 61% of all LA residents are fan of LA Lakers.

$\hat{p} = \frac{.61 + .51}{2} = \boxed{.56}$

$E = \frac{.61 - .51}{2} = \boxed{.05}$

Nov 30-6:37 AM

7.5% of 140 randomly Selected students were left-handed.

$n=140$

$\hat{p}=.075$ $\Rightarrow x = n\hat{p} = 140(.075) = 10.5$

$\boxed{x=11}$

Find 99% conf. interval for the prop. of all students that are left-handed.

$\rightarrow$ C-level: .99

1-Prop ZInt

$x=11$

$n=140$

C-level: .99

$.02 < p < .13715$

Final Ans. $\rightarrow$ $\boxed{.02 < p < .14}$

we are 99% confident that between 2% and 14% of all students are left-handed.

$\hat{p} = \frac{.14 + .02}{2} = \boxed{.08}$

$E = \frac{.14 - .02}{2} = \boxed{.06}$

Nov 30-6:47 AM

Now

Constructing Confidence Interval for population mean:

Ans: $<\mu<$

Format: $\bar{x}-E<\mu<\bar{x}+E$

Sample Mean

Point- estimate

Margin of error

Case I: σ Known	Case II: σ unknown
$E=Z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}$	$E=t_{\alpha/2}\cdot\frac{S}{\sqrt{n}}$ $\to df=n-1$
STAT TESTS $\boxed{\text{Z Interval}}$	STAT TESTS $\boxed{\text{T Interval}}$
inpt: $\boxed{\text{Stats}}$	Inpt: $\boxed{\text{stats}}$

Nov 30-7:10 AM

Given: $\boxed{n=36}$, $\boxed{\bar{x}=84}$, $\boxed{\sigma=10}$, $\boxed{\text{C-level}: .98}$

Find conf. interval for pop. mean.

Since σ Known

Z Interval

inpt: $\boxed{\text{stats}}$

$\sigma=10$

$\bar{x}=84$

$n=36$

c-level: .98

$80.123<\mu<87.877$

Since $\bar{x}$ is a whole #

$\boxed{80<\mu<88}$

We are 98% Confident that mean of all is between 80 & 88.

$\bar{x}=\frac{88+80}{2}=\boxed{84}$

$E=\frac{88-80}{2}=\boxed{4}$

Nov 30-7:16 AM

Given: $n=16$, $\bar{x}=125.5$, $S=20.8$

C-level $=.9$ $\longrightarrow$ $df = n-1 = 15$

Find Conf. interval for population mean.

Since σ unknown

T Interval

inpt: $\boxed{\text{STATS}}$

$\bar{x}=125.5$

$S=20.8$

$n=16$

C-level: .9

$\bar{x}=\frac{134.6+116.4}{2}=\boxed{125.5}$

$E=\frac{134.6-116.4}{2}=\boxed{9.1}$

$116.38<\mu<134.62$

Since $\bar{x}$ was in 1-decimal, we round to 1-decimal.

$\boxed{116.4<\mu<134.6}$

We are 90% Confident that mean of all is between 116.4 & 134.6.

Nov 30-7:23 AM

In a Survey of 40 nurses, their mean monthy Salary was \$6175. $n=40$ $\bar{x}=6175$ Point-estimate

It is known that standard deviation of monthly Salaries of all nurses is \$425. $\sigma=425$

Find Conf. interval for the mean monthly Salary of all nurses.

No C-level ⇒ USE .95

Since σ Known, use

Z Interval

inpt: $\boxed{\text{STATS}}$

$\sigma=425$

$\bar{x}=6175$

$n=40$

C-level: .95

$\bar{x}=\frac{6307+6043}{2}=\boxed{6175}$

$E=\frac{6307-6043}{2}=\boxed{132}$

$6043.3<\mu<6306.7$

Since $\bar{x}$ is a whole #, we round to whole #

$\boxed{6043<\mu<6307}$

We are 95% Confident that the mean monthly Salary for all nurses is between \$6043 and \$6307.

Nov 30-7:31 AM

I randomly selected $\boxed{10 \text{ exams}}$, here are the scores:

75	82	100
65	78	95
80	70	90
	58	

$n = 10 \longrightarrow df = n - 1 = \boxed{9}$

Find

1) $\bar{x} = 79.3$

2) $S = 13.2$

Round to 1-decimal

3) Find 99% conf. interval for the mean of all exams

C-level: .99

σ unknown

T Interval

inpt: $\boxed{\text{Stats}}$

$\bar{x} = 79.3$

$S = 13.2$

$n = 10$

C-level: .99

$65.735 < \mu < 92.865$

Since $\bar{x}$ is 1-decimal, we round to 1-decimal

$\boxed{65.7 < \mu < 92.9}$

$\bar{x} = \frac{92.9 + 65.7}{2} = \boxed{79.3}$

$E = \frac{92.9 - 65.7}{2} = \boxed{13.6}$

Nov 30-7:42 AM

Find $t_{\alpha/2}$ for 90% C-level with $df = 15$.

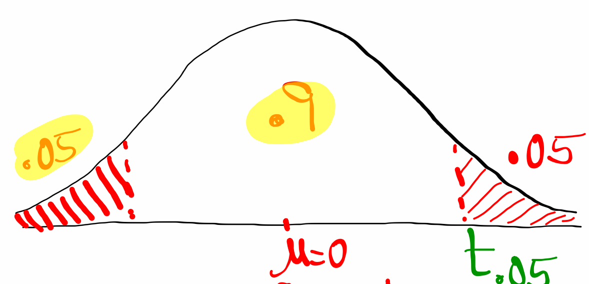
t-dist

$\underbrace{90\% \text{ C-level}}_{\text{middle Area}}$

.9

$1 - .9 = .1 \leftarrow \alpha$

$.1 \div 2 = .05 \leftarrow \alpha/2$



σ unknown

$df = 15$

$t_{.05} = \text{invT}(.95, 15) = \boxed{1.753}$

Nov 30-7:52 AM

Find $t_{\alpha/2}$ for $\alpha = .05$ and $df = 9$.

$1 - \alpha = .95$ ← Middle Area 95% C-level

$\alpha/2 = .025$ ← Area on each tail

σ unknown
df=9

$t_{.025} = \text{invT}(.975, 9) = \boxed{2.262}$

Nov 30-7:56 AM

What is degrees of freedom?

There are 10 students.

I bring 10 donuts for them.

First person → 10 choices

Second " → 9 choices

Third " → 8 "

⋮

Last person → No choices (1 donut left)

9 had choices out of 10 people

$\boxed{df = 9}$

Nov 30-7:59 AM

7 clean shirts

Monday → 7 choices

Tuesday → 6 "

Wednesday → 5 "

⋮

Sunday → No choices (1 clean shirt)

$df = 7 - 1 = \boxed{6}$

6 days you had choices. Not on last day.

Thursday Plan:

lecture → 6:00 – 6:45

Exam II → 6:50 – 9:00

Nov 30-8:04 AM